BOOK REVIEW *

Quantum Measurement of a Single System
by Orly Alter and Yoshihisa Yamamoto

reviewed by Matthew J. Donald

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O. Alter and Y. Yamamoto, *Quantum Measurement of a Single System* (Wiley, 2001), 136 pp, ISBN 0-471-28308-8.

This is an interesting but rather brief monograph which is beautifully produced and amusingly illustrated. In it, Alter and Yamamoto provide a series of analyses of specific models of measurements, and sequences of measurements, on single quantum systems. They use these analyses to comment on a number of controversial issues and to discuss their central theme of the limitations on the information about the wavefunction of a single system which can be gained by measurement. Their analyses are mainly clear and careful, although a significant amount of work is required to follow the details. However, I do find somewhat problematic their tendency to generalize from specific models to universal statements. Some of these statements might give a misleading impression of the book as a general treatment of questions in quantum measurement theory; but it would be more appropriate to think of it as a short and well-organized discussion of some novel and important examples together with useful pointers to a wider literature.

According to elementary quantum mechanics, each quantum system has an associated wavefunction at any time. "Measuring" such a system involves choosing an observable (a self-adjoint operator) and then performing some physical process as a result of which the wavefunction of the system "collapses" to one of the eigenfunctions of the observable with probabilities determined, by the Born rule, from the original wavefunction. At every point, this account has been the subject of fierce argument and proposed revision. Alter and Yamamoto's version merely adds one additional stage. In their models, the "physical process" is taken to be a specific quantum interaction with a quantum "probe". Subsequent collapse of the probe wavefunction then entails a corresponding collapse in the system wavefunction. Even this apparently minor step, which after all does nothing to solve the "measurement problem", provides a framework in which a wide range of ideas can be tested. In particular, it allows the analysis of sequences of weak or imprecise measurements on a single quantum system.

In elementary quantum mechanics, a single measurement of observable Q on a single system with wavefunction ψ results with probability $|<\psi|\psi_q>|^2$ in collapse to

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an eigenfunction ψ_q with corresponding eigenvalue q. It is then clear that, trivial cases apart (i.e. as long as $\psi \neq \psi_q$), we have irretrievably lost some of the information in ψ. In particular, although in the value q we gain an estimate of $<\psi|Q|\psi>$, we learn nothing about the corresponding variance: $<\psi|Q^2|\psi> - <\psi|Q|\psi>^2$.

Alter and Yamamoto's first detailed model is of a sequence of photon-number measurements on a single squeezed state of light. In each measurement, the measured system (or signal state) is correlated to a probe state in an optical Kerr medium. Assuming appropriate Gaussian probability densities in the states of signal and probes, the change in the signal state on each measurement can be explicitly calculated, as can the probability distribution of the measured results. It is therefore possible to compare a sequence of imprecise measurements with a single precise measurement. Alter and Yamamoto demonstrate that both allow estimation of the original photon-number but that neither provide information about the original photon-number variance.

In Alter and Yamamoto's two-stage analysis, it is collapse of the probe state which leads, by entanglement, to collapse of the signal state. They therefore move on to contemplate measurements in which the signal and the probe are unentangled at the end of their interaction. Again, they analyse a specific solvable model; in this case, the measurement of a squeezed harmonic oscillator signal state coupled linearly to a squeezed vacuum probe state. They demonstrate that the output signal and probe are disentangled only when there is a particular relation between their squeezing parameters. This implies that for these measurements to be made without entanglement, it is necessary for the probe to be prepared with properties which match those of the signal and so those properties must already be known to the experimenter.

Next Alter and Yamamoto turn to the idea of a "protective measurement". As with many of the topics they introduce, their discussion here is too brief to amount to a full review. Nevertheless, they do provide ample references to both sides of their arguments. Their main contribution to the debate consists of the analysis of yet another exact solution to an appropriate model. This model they use to criticize approximations which have suggested that "adiabatic measurements" can be made without resulting in entanglement between probe and signal.

In discussing the effect of measurement on the unitary time evolution of a single system, Alter and Yamamoto demonstrate an equivalence between a sequence of measurements designed to discover that evolution and a sequence of measurements designed to discover the initial state of the system without the unitary evolution. This indicates that limits on our ability to measure the initial states of single systems imply limits on our ability to measure their time evolutions. Alter and Yamamoto illustrate this with a model of measurements of photon number for a two-level atom in a single mode cavity.

The final topic of the book is the monitoring of a single quantum system driven by an unknown classical force. Here, although Alter and Yamamoto do provide some useful analysis, there are also some errors, as, for example, in the sentence including equation (7.9) in which it would appear, if equation 7.2 is invoked, that the uncertainty principle is violated.

At the conclusion of their book (page 118), Alter and Yamamoto claim that "the information that can be obtained about the quantum wavefunction of a single system in a series of measurements of this system cannot account for the physical reality (i.e. ontological meaning) of the wavefunction, and that the quantum wavefunction is limited to having a statistical (i.e. epistemological) meaning only". Their claim on page 6 is quite unambiguous: "the quantum wavefunction cannot be ascribed physical reality". In my opinion, however, Alter and Yamamoto have failed to justify these claims. Indeed, I think that it would hardly even have helped their case, although it would have improved their book, if, for example, rather than merely providing specific models, they had explained the general arguments put forward in the paper they cite by D'Ariano and Yuen (Phys. Rev. Lett. **76**, 2832–2835, (1996)). The problem, however, is not that I think that there are any situations in which, with no prior knowledge, it *would* be possible to determine the wavefunction of a single system by a series of measurements but rather that, it seems to me, that to make the case that wavefunctions do not have physical reality does require at least some explanation of what else might.

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