

BOOK REVIEW \*

**Quantum Evolution**  
by Johnjoe McFadden

reviewed by Matthew J. Donald

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J. McFadden, *Quantum Evolution: Life in the Multiverse* (HarperCollins, 2000), 338 pp, ISBN 0-00-255948-X, 0-00-655128-9.

In “Quantum Evolution”, Johnjoe McFadden makes far-reaching claims for the importance of quantum physics in the solution of problems in biological science. In this review, I shall discuss the relevance of unitary wavefunction dynamics to biological systems, analyse the inverse quantum Zeno effect, and argue that McFadden’s use of quantum theory is deeply flawed.

In the first half of his book, McFadden both discusses the biological problems he is interested in solving and gives an introduction to quantum theory. This part of the book is excellent popular science. It is well-written, competent, and fun.

As far as the biology is concerned, McFadden, who is a molecular microbiologist, has very specific, and often controversial, opinions. Nevertheless, he does refer to a wide range of alternative points of view. He certainly managed to convince me that I had swallowed too easily the prevailing dogma (as found, for example, in chapter 1 of [Albert et al. 1989](#)) about the earliest (“prebiotic”) stage in biological evolution during which the first self-replicating molecules appeared. McFadden argues that this stage seems to have happened quite fast in terms of the age of the Earth (perhaps within 100 million years) but that none of the proposed mechanisms, of which he discusses several, give anything like a complete and plausible picture of how to go from the early Earth’s chemistry to the first cell. Rescuing the prevailing dogma would require a suitable sequence of laboratory experiments, or, at the very least, plausible computer simulations. Unfortunately, while 100 million years is quite short in the age of the Earth, it is rather long in the laboratory.

When McFadden moves on to describe quantum mechanics and its interpretations, I feel that his touch becomes rather less sure. However it is hardly surprising that I can tell that he is out of his primary field and into mine, and the outlines of his presentation still strike me as reasonable. Unfortunately it is the details which matter when he attempts to apply quantum theory. He is not sufficiently explicit about what

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\* [quant-ph/0101019](#), January 2001. In [quant-ph/0110083](#), Johnjoe McFadden and Jim Al-Khalili reply to this review. My response to their reply is at <http://people.bss.phy.cam.ac.uk/~mjd1014/qevrevr.html>

is going on at the level of the quantum state. In my opinion, this eventually leads him hopelessly astray.

As well as the appearance of self-replication, McFadden is interested in the possibility of “adaptive” or “directed” mutation. This is the claim, reviewed by [Lenski and Mittler \(1993\)](#), that there is some evidence that, in some bacteria, some mutations will tend to appear more frequently in circumstances when they are advantageous than when they are biologically neutral. McFadden also sketches a version of the idea that free will is a quantum phenomenon. In all three cases, the technical core of McFadden’s proposals involves the quantum dynamics of molecular systems.

The analysis of quantum dynamics is not entirely straightforward. According to the traditional account, the wavefunctions of quantum systems change in two quite different ways. Sometimes there is abrupt, indeterministic, change – “wavefunction collapse” – described by the “projection postulate” in which, with a probability given by a squared transition amplitude, the wavefunction is replaced by some eigenfunction of a “measurement operator”. At all other times, the wavefunction changes according to a Schrödinger equation, so that the dynamics is defined by a unitary group of the form  $U(t) = \exp(-itH)$  with  $H$  a self-adjoint Hamiltonian operator.

More modern accounts invoke decoherence theory ([Giulini et al. 1996](#)) in order to justify the assumption that there is always some sufficiently large scale on which the dynamics is unitary. In particular, decoherence theory encourages us to treat “measurements” as physical processes governed by appropriate global Schrödinger equations and to interpret the abrupt changes as merely the way in which changes in one part of the global quantum system would appear to another part.

Although decoherence theory does leave open crucial conceptual problems in the interpretation of quantum theory, it suggests that those problems are not primarily matters of dynamics. The Schrödinger equations which apply in biological situations are precisely those of conventional quantum chemistry and are defined ultimately by the electromagnetic interactions between electrons and nuclei in a fixed background classical gravitational field. Certainly if, as he claims, McFadden’s ideas are compatible with the many-worlds interpretation of quantum theory, then we should also be able to analyse his work in terms of global unitary non-relativistic electromagnetic quantum dynamics. In general, however, subsystems of a system with unitary dynamics will not themselves have unitary (reversible) dynamics unless they are effectively isolated from their surroundings. Moreover, subsystems of a system in a pure (wavefunction) state will not necessarily themselves occupy pure states. Indeed, any account of local states entirely in terms of product wavefunctions, even if it is possible at a single instant, will disregard important issues of thermal physics which are particularly relevant to the apparent dynamics of biological systems.

Consider then McFadden’s proposals about the process by which self-replicating proteins appear. (McFadden does mention that according to the prevailing dogma self-replication in RNA arose before self-replication in proteins, but he argues that RNA is completely implausible as a prebiotic chemical.) Assume that the early Earth

produces a sea of amino acids which can link into peptides a few of which are self-replicating. According to McFadden, the chance of a random peptide being self-replicating is far too small for this process to be a plausible source of life in the framework of classical biochemistry. He claims that, instead, we must invoke what he calls the “inverse quantum Zeno effect”. (Thorough introductions to quantum Zeno effects are given in chapter 8 of [Namiki, Pascazio, and Nakazato 1997](#) and in section 3.3.1 of [Giulini et al. 1996](#).)

The idea of the inverse quantum Zeno effect goes back to [von Neumann \(1932, section V.2\)](#). For a mathematician, it is some version of [the theorem](#) proved in the appendix which shows that, up to analytical niceties, if the projection postulate is true, and if we can choose our measurements at will, then, by using the right dense sequences of measurements, we can turn some given pure state  $|\Phi\rangle\langle\Phi|$  into any other pure state  $|\Psi\rangle\langle\Psi|$  regardless of the physical Hamiltonian.

In order to prove this, we first need to use some form of the mathematically-simple existence lemma [\(A.1\)](#) which states that there exists a bounded self-adjoint Hamiltonian  $K$  such that  $\exp(-iK)\Phi = \Psi$ . However, the problem with such an existence lemma is that  $K$  is totally artificial. It is a purely mathematical adjunct required to reduce to the theorem for the standard Zeno effect in which we have  $\Psi = \Phi$ .

The crucial fact on which the inverse Zeno effect depends, is that, for any pair of unitary propagators  $U(t)$  and  $V(t)$  defined by suitable Hamiltonians,

$$|\langle\Phi|U(1/N)V(-1/N)|\Phi\rangle|^2 = 1 - O(1/N^2). \quad (1)$$

In [theorem A.2](#), this fact appears as [\(A.3\)](#) with  $V(t) = e^{-itK}$ .

(1) is an extension of the principle that unitary change in quantum theory begins slowly. Indeed, using  $N \times (1/N^2) = 1/N$ , the idea is that if we can interrupt a change sufficiently often then we can gradually alter it in any given direction. To make use of this, however, we actually do have to have large numbers of interruptions applied within a bounded time. It should also be noted that the interruptions, or projections, constructed in the proof are strongly dependent, through [lemma A.1](#), on the initial and final wavefunctions.

Decoherence theory tells us that the projection postulate can be viewed as a phenomenological aspect of a unitary dynamics at a large scale. For a preliminary version of how this works at a mathematical level, we can use another existence lemma similar to [lemma A.1](#). What [lemma A.4](#) means in words is that the projection postulate can always be modelled as the restriction of a unitary dynamics on a large space to density matrix (or mixed state) dynamics on a pair of smaller spaces.

Once again, however, the Hamiltonian  $L$  in [lemma A.4](#) is totally artificial. For each of the  $N$  projections  $P_n^N$  in [theorem A.2](#) we need to have access to an auxiliary space. We also require that the time  $s$  for which the corresponding map  $\exp(-isL)$  should apply should satisfy  $s \sim 1/N$  and that the dynamics should be switched off at the end of that brief period. The famous experiments of [Itano et al. \(1990\)](#) involved not only a series of brief interactions with auxiliary spaces of photon states, but also a system in which there was an essentially thermodynamical reason for rapid relaxation

onto the projected state. Thus there were very specific constraints on the form of the physical dynamics and on the projections.

As a physical example of the inverse quantum Zeno effect, McFadden considers three polarizing lenses. Adjust lenses 1 and 2 to block light entirely. Then insert lens 3 between lenses 1 and 2. With lens 3 turned to a suitable angle, some light will pass through the entire sequence.

The simple analysis in the appendix is not entirely relevant to this case, because our observations of light intensity involve the average, macroscopic, behaviour of many photons. Nevertheless, this is an excellent example. It shows that the Zeno effect is not magic. We are dealing here with a situation in which it is possible to understand why the projection postulate should be a reasonable model of the interaction between the measured property of the light and the lens. The measurements involved are genuine and identifiable physical processes. Nevertheless, the required sequence of measurements has to be carefully constructed and adjusted. The light polarization to which the effect is applied is a particularly simple and easily controlled property. Moreover, when the experiment is performed, one sees, in the inefficiency of light transmission through a real lens, the difference between the mathematical ideal and what is possible in practice.

There are many ways in which evolution has become very good at careful construction and adjustment, but this was certainly not true at the prebiotic stage. As a mechanism for producing desirable peptides on a lifeless planet, my opinion is that the inverse Zeno effect has no plausibility. I would only change that opinion if I was presented with a detailed model, at the level of the quantum states of all the particles involved. The essential difficulty is to explain why the actual molecular dynamics should have the slightest resemblance to what is merely a theoretically possible dynamics.

Instead of giving such an explanation, McFadden, both in his book and in a published paper (McFadden and Al-Khalili 1999) on adaptive mutation, provides a much vaguer picture of molecular systems as sometimes being isolated and sometimes decoherent, and he suggests that when the systems he considers come out of isolation, suitable measurements will drive them towards desirable biological goals. This picture has much in common with the idea of a quantum computer. As such of course we can return to the necessity for experiments. Here the primitiveness of our artificial quantum computers and the difficulty that we have in building them, seems to me to be fairly good evidence that the sort of multi-atom wavefunction control which would be required to make McFadden's ideas work is unlikely to have occurred by chance.

At the prebiotic stage, McFadden wants to consider a soup of as many as 20 varieties of amino acid with different side chains as being isolated "inside tiny structures: perhaps in the pores of a rock or within a chemically generated oil or protein droplet". Amino acid residues vary in volume from  $60 \text{ \AA}^3$  for glycine to  $228 \text{ \AA}^3$  for tryptophan. They vary in structure by as many as seventeen atoms and they vary in pH and in all sorts of other reactivities (Creighton 1983). I find it inconceivable that in a peptide in a drop of warm wet fluid we could exchange a tryptophan residue for a glycine residue without the state of every molecule in the drop rapidly being affected.

After all, the change of volume in the peptide would provide room for more than five water molecules.

In their analysis of adaptive mutation, McFadden and Al-Khalili consider an entire bacterial cell as an isolated unit with a possible mutation resulting in a protein in which an arginine residue is replaced by a histidine residue. In comparison to other residue pairs, arginine and histidine are fairly similar. They are both bases, and they differ in volume by only about 2/3rds of a water molecule. Nevertheless, their structures are still substantially different, with arginine containing one more nitrogen and five more hydrogen atoms.

McFadden and Al-Khalili produce the following equation to describe the wavefunction of a cell in this situation:

$$|\Psi_{cell}\rangle = \alpha|\Phi_{not\ tun.}\rangle|C\rangle|Arg\rangle + \beta|\Phi_{tun.}\rangle|T\rangle|His\rangle. \quad (2)$$

Here  $\Phi$  is supposed to denote the wavefunction of a proton on a gene which can tunnel between two possible positions.  $|C\rangle$  and  $|T\rangle$  denote corresponding wavefunctions of a daughter DNA strand following replication and  $|Arg\rangle$  and  $|His\rangle$  denote the wild-type and mutant form of the protein.

At best, equation (2) can be considered as schematic, ignoring as it does all the other particles which will be affected by the processes of DNA replication and transcription and protein synthesis. Indeed, as they refer to different numbers of particles, neither “ $|C\rangle$ ” and “ $|T\rangle$ ” nor “ $|Arg\rangle$ ” and “ $|His\rangle$ ” can even be considered to belong to the same Hilbert spaces. This is not just poor notation; except in special circumstances like superconductivity, the more particles are involved in a situation, the less plausible coherence becomes. As I shall discuss further below, there is no justification for the assignment of a pure state over an extended period to a system like a bacterial cell. This difficulty will extend even to quite small parts of the cell, including single proteins and nucleic acids with their complicated internal thermal mobilities (McMammon and Harvey 1987). It will certainly extend to the groups of molecules necessarily involved in the two components of equation (2). As a general rule, coherence is destroyed at least as easily as the most minute quantities of heat are passed. In a fluid system, heat is rapidly exchanged not only between the translational degrees of freedom of separate molecules but also between internal rotational and vibrational degrees of freedom.

McFadden and Al-Khalili suppose that, in the absence of a substrate on which the cell can grow, the coherence expressed by equation (2) can persist over times (1 – 100 s) of biological significance. They attempt to justify this claim by referring to measurements of NMR  $T_1$  spin-lattice relaxation times for protons in biological systems. This is an error not only because equation (2) involves far more extensive state changes than the motion of a single proton, but also because it is much easier for heat to pass between position degrees of freedom than to or from nuclear spins. They then suppose that if the cell is provided with a substrate for which the mutated protein “His” is adaptive, the probability of the second term in the wavefunction will somehow be enhanced. They appear to claim that this will happen simply because, in the presence of an adaptive substrate, the decoherence time will decrease. Unfortunately, because “His” is the mutated and rarer form of the protein, we should assume in

normal circumstances that  $|\beta| < |\alpha|$  and that, for short times  $t$ , at least in the simplest model,  $|\beta|^2 \sim \sin^2 \lambda t \sim \lambda^2 t^2$ . This makes mutation analogous to decay of a quantum energy level. But the normal idea is that “observation”, or at least the standard quantum Zeno effect, should slow down decay (albeit, usually to an imperceptibly small extent) rather than speed it up. Thus McFadden and Al-Khalili really do require something like the inverse quantum Zeno effect with its precisely adjusted interactions. Although such interactions might exist in principle at a mathematical level, I see no reason to believe that biological components can provide them; either spontaneously, or by natural evolution, or by human construction.

Quantum theory is undoubtedly important for the complete understanding of the dynamics of biological systems for reasons which go somewhat beyond the relevance of quantum chemistry in all detailed studies of molecular interactions. In particular, some electron transfer systems, such as those involved in photosynthesis, can only be properly described by a detailed quantum-statistical-mechanical analysis (DeVault 1984, Kilin 2000). Photosynthesis itself demonstrates that it is possible for biological systems to evolve to capture individual photons. The resulting systems are wonderfully complex and it seems that they could only have arisen after a long history of refinement of membrane-bound electron transfer systems (Albert et al. 1989, chapter 7). Even so, excitation of an electron on one molecule, and transfer of that electron to a succession of different molecules involves the ordinary building blocks of biology; collisions between specific molecules in the right environmental conditions. The execution of an arbitrary sequence of mimics of the projection postulate would require an entirely different level of wavefunction control.

Ogryzko (1997) also attempts to explain adaptive mutation in quantum mechanical terms. His suggestion is that some kind of environment-dependent measurement process is involved. Like McFadden and Al-Khalili, Ogryzko attempts to extend a physical theory, appropriate for some simple systems, way beyond its normal domain of application. Here too, my opinion is that the proposal has no plausibility, and challenge Ogryzko to produce an analysis which is explicit about the form of his “measurement operators”.

McFadden mentions the anthropic principle as an alternative way of dealing with the apparent improbability of the first step in the evolution of life. The argument here is that an event of any non-zero probability, however low, will occur somewhere within the “multiverse” of many-worlds theory. Thus we might observe a past in which a self-replicator came into being in a random sequence of peptide linkages, and we inevitably will observe such a past, if it is the only way to explain our existence as observers. McFadden rejects this argument, both because he “would dearly like to believe that we are not alone in the universe”, and because, as he points out, the argument fails to explain the crucial issue, which is how self-replicators can have plausibly arisen within such an early period of the Earth’s existence. If the only way that observers could exist was through an extremely unlikely quantum event followed by biological evolution, then we would expect our universe to have lots of planets; so that it would be very big (which it may well be), and/or very old (which it probably is not). We would also expect that, when the quantum event happened,

it would happen at a typical time within the life of the planet on which it happened, as long as there was still time for the required subsequent evolution. As the expected main sequence life of the Sun is around ten billion years, while evolution has occurred within four billion years, we would therefore expect to see geological evidence of a several billion year period during which the Earth was lifeless. On the other hand, if we were prepared to make the argument using self-replicators from space, then we would expect our observed universe to have lived for far more star lifetimes than it apparently has.

In the final chapter of his book, McFadden turns his attention to the brain. Here he suggests that the neural electromagnetic field might be the seat of consciousness. Personally, I fail to understand why he, or anyone else, should believe that such a spatially-extended field can solve the problem of the unity of consciousness in any way that a spatially-extended pattern of neural firings cannot. He confuses the quasi-classical electro-magnetic field with the wave function of the quantization of that field. He also speaks of single photon events in the circumstances of voltage-gated ion channel opening in which there is no reason to suppose that the state moves between eigenstates of a photon number operator.

In my opinion, the most fundamental mistake in McFadden's work is the assumption that unitary wavefunction dynamics can provide an accurate description of the behaviour of systems such as entire cells over significant time periods. The dynamics implied by this assumption is incorrect under any interpretation of quantum mechanics. An interpretation in which a cell did have a wavefunction at all times would require extremely frequent wavefunction collapse, contradicting unitarity.

Detailed arguments against mistakes of this kind usually involve models of the effect of scattering processes on quantum states (Joos and Zeh 1985, Giulini et al. 1996 Section 3.2, Tegmark 2000). Defence against such arguments require analysis of lengthscales and timescales, of the details of interactions, and of the type of quantum system within which coherence is claimed (Hagan, Hameroff, and Tuszynski 2000). At the micron scale of a bacterial cell, coherence is only observed in very special systems such as lasing light and superconducting electrons. In the first case, we have self-reinforcement (or "amplification") of the macroscopic quantum properties, and in the second, we have a multiply-occupied ground state isolated by an energy gap. In both cases, as in the case of light polarization considered earlier, the macroscopic nature of the situation is neutralized by macroscopic uniformity and by the simplicity of the interactions of certain degrees of freedom.

Careful isolation and state preparation are the hallmarks of experimental science and can be found in most cases for which the projection postulate provides a good model of the ultimate observations. A fascinating recent example comes from the diffraction of fullerene molecules (Arndt et al. 1999). In this case, coherence is observed in the centre-of-mass of hot complex molecules. State preparation is by careful collimation. Isolation, both from internal and external influence, is for a short flight in an evacuated chamber. Controlled state preparation for a system as complex as a bacterial cell is essentially impossible, while in order to produce the required isolation

over the required period, the cell would have to be placed in a vacuum which would be fatal to it.

On the other hand, there are also examples in which entirely natural systems do exhibit unitary wavefunction dynamics over indefinite periods. For example, there can be no doubt that it is appropriate to assign a stationary pure state to the core electrons of an atom inside a biological system. Here the isolation is due to the large quantity of energy which would have to be supplied in order to free the electrons. The state preparation is thermodynamical. The nucleus would have been unable to trap the electrons had they not lost energy to the electromagnetic field.

Beyond these various special systems, it is possible that wavefunction dynamics may be found at the level of the entire universe. Otherwise, we are always entitled to require detailed and specific models and explanations of why entropy and temperature are not relevant parameters in the observed behaviour. Biological systems, in particular, are fundamentally warm and wet. Of course cells are precisely structured at the level of the compartments into which they divide their various functions. However, at the molecular level within those compartments, or on the membranes which bound them, heat not only keeps life moving, but also provides energy to overcome activation barriers and dissipation to maintain irreversibility. Rotating and vibrating, molecules roll and rock from collision to collision until, when they find the right partners and the right enzyme, they find a free energy gradient down which they can fall (Harold 1986). The partners are transformed and their successors wander off for new adventures.

A correct quantum theoretical analysis of biological systems has to take account of the local free energy gradients which drive them. This implies that our descriptions of most of the degrees of freedom relevant to the functioning of biological systems should involve mixed states and chance. In many ways this simplifies the description of the dynamical processes involved; classical biochemistry provides a precise picture of the typical dynamics. Nevertheless, at some point, we need to investigate the fundamental meaning of quantum theoretical chances and descriptions. This brings us back to the crucial open conceptual questions of decoherence theory. My own opinion (Donald 1990, 1999) is that the study of biological systems may be no less important in answering these questions than is the study of quantum theory for a complete understanding of the observed dynamics of biological systems.

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## Appendix

**lemma A.1** Let  $\mathcal{H}$  be a Hilbert space  $\mathcal{H}$  and  $\Phi, \Psi \in \mathcal{H}$  with  $\|\Phi\| = \|\Psi\| = 1$ .

Then there exists a bounded self-adjoint operator  $K$  on  $\mathcal{H}$  such that  $e^{-iK}\Phi = \Psi$ .

*proof* Define  $\delta \in [0, 2\pi)$  by  $\langle \Phi | \Psi \rangle = e^{-i\delta} |\langle \Phi | \Psi \rangle|$  with  $\delta = 0$  if  $\langle \Phi | \Psi \rangle = 0$ .

If  $|\langle \Phi | \Psi \rangle| = 1$  then  $\Psi = e^{-i\delta} \Phi$ , so choose  $K = \delta$ .

Otherwise  $|\langle \Phi | \Psi \rangle| < 1$  and  $\Phi_\perp = \frac{e^{i\delta} \Psi - |\langle \Phi | \Psi \rangle| \Phi}{\sqrt{1 - |\langle \Phi | \Psi \rangle|^2}}$  satisfies  $\langle \Phi | \Phi_\perp \rangle = 0$  and

$\langle \Phi_\perp | \Phi_\perp \rangle = 1$ .

Set  $\sin \theta = \sqrt{1 - |\langle \Phi | \Psi \rangle|^2}$  with  $0 \leq \theta \leq \frac{\pi}{2}$ . Then  $\cos \theta = |\langle \Phi | \Psi \rangle|$  and so  $e^{i\delta} \Psi = \Phi \cos \theta + \Phi_\perp \sin \theta$ .

Now let  $K' = -i\theta |\Phi\rangle\langle\Phi_\perp| + i\theta |\Phi_\perp\rangle\langle\Phi|$  and  $K = K' + \delta$ .

$-iK' = -\theta |\Phi\rangle\langle\Phi_\perp| + \theta |\Phi_\perp\rangle\langle\Phi|$  and  $(-iK')^2 = -\theta^2 (|\Phi\rangle\langle\Phi| + |\Phi_\perp\rangle\langle\Phi_\perp|)$ .

Thus  $(-iK')^{2N} = (-1)^N \theta^{2N} (|\Phi\rangle\langle\Phi| + |\Phi_\perp\rangle\langle\Phi_\perp|)$

and  $(-iK')^{2N+1} = (-1)^N \theta^{2N+1} (-|\Phi\rangle\langle\Phi_\perp| + |\Phi_\perp\rangle\langle\Phi|)$  and so

$$e^{-iK'} \Phi = \sum_{N \geq 0} \frac{(-iK')^N}{N!} \Phi = \Phi \cos \theta + \Phi_\perp \sin \theta = e^{i\delta} \Psi. \quad \blacksquare$$

**theorem A.2** Let  $H$  be a self-adjoint operator on a Hilbert space  $\mathcal{H}$ . Let  $Q_m$  be the spectral projection of  $H$  for the interval  $[-m, m]$ . Suppose  $\Phi, \Psi \in \cup_{m=1}^\infty Q_m \mathcal{H}$  are normalized. Write  $U(t) = e^{-itH}$ .

Then there exist sequences  $(P_n^N)_{n=1}^N$  of projections such that,

$$|\Psi_N\rangle\langle\Psi_N| \rightarrow |\Psi\rangle\langle\Psi| \text{ as } N \rightarrow \infty$$

$$\text{where } \Psi_N = P_N^N U(1/N) P_{N-1}^N U(1/N) \dots U(1/N) P_2^N U(1/N) P_1^N U(1/N) \Phi.$$

*proof* Let  $K$  be the bounded self-adjoint operator constructed in lemma A.1 with  $e^{-iK}\Phi = \Psi$ . Note that we can choose  $M$  such that  $\Phi, \Psi \in Q_M \mathcal{H}$ , and then, from the proof of lemma A.1, for all  $t$ ,  $e^{-itK}\Phi \in Q_M \mathcal{H}$ .

Define  $P_n^N = e^{-inK/N} |\Phi\rangle\langle\Phi| e^{inK/N}$ .

As  $\|\Psi\| = 1$  and  $\|\Psi_N\| \leq 1$ , to prove that  $|\Psi_N\rangle\langle\Psi_N| \rightarrow |\Psi\rangle\langle\Psi|$ , it is sufficient to prove that  $|\langle\Psi|\Psi_N\rangle|^2 \rightarrow 1$  or that  $\log |\langle\Psi|\Psi_N\rangle|^2 \rightarrow 0$ .

$$|\langle\Psi|\Psi_N\rangle|^2 = \prod_{n=1}^N |\langle\Phi|e^{inK/N} e^{-iH/N} e^{-i(n-1)K/N} |\Phi\rangle|^2.$$

Let  $|\Phi_n\rangle = e^{-inK/N} |\Phi\rangle$  and

$$a_n(t) = |\langle\Phi_n|e^{-itH} e^{itK} |\Phi_n\rangle|^2 = \langle\Phi_n|e^{-itK} e^{itH} |\Phi_n\rangle \langle\Phi_n|e^{-itH} e^{itK} |\Phi_n\rangle.$$

$a_n(t)$  is infinitely continuously differentiable.  $a_n(0) = 1$ ,

$$\begin{aligned} a'_n(t) &= i(\langle\Phi_n|e^{-itK} (H - K) e^{itH} |\Phi_n\rangle \langle\Phi_n|e^{-itH} e^{itK} |\Phi_n\rangle \\ &\quad + \langle\Phi_n|e^{-itK} e^{itH} |\Phi_n\rangle \langle\Phi_n|e^{-itH} (K - H) e^{itK} |\Phi_n\rangle), \end{aligned}$$

$$a'_n(0) = 0,$$

$$a''_n(t) = -(\langle \Phi_n | e^{-itK} (H^2 - 2KH + K^2) e^{itH} | \Phi_n \rangle \langle \Phi_n | e^{-itH} e^{itK} | \Phi_n \rangle \\ + 2\langle \Phi_n | e^{-itK} (H - K) e^{itH} | \Phi_n \rangle \langle \Phi_n | e^{-itH} (K - H) e^{itK} | \Phi_n \rangle \\ + \langle \Phi_n | e^{-itK} e^{itH} | \Phi_n \rangle \langle \Phi_n | e^{-itH} (K^2 - 2HK + H^2) e^{itK} | \Phi_n \rangle),$$

$$\text{and } |a''_n(t)| \leq 4(M + \|K\|)^2.$$

Thus

$$a_n(t) = 1 + \int_0^t a'_n(u) du = 1 + \int_0^t \int_0^u a''_n(v) dv du$$

$$\text{and so } |a_n(t) - 1| \leq 2t^2(M + \|K\|)^2. \quad (\text{A.3})$$

For  $\frac{1}{2} \leq x \leq 1$ ,  $0 \leq -\log x = \int_x^1 \frac{1}{y} dy \leq 2(1-x)$  and so, for  $N$  sufficiently large,

$$0 \leq -2 \log |\langle \Psi_N | \Psi \rangle| = - \sum_{n=1}^N \log a_n(1/N) \leq \sum_{n=1}^N 4(1/N)^2 (M + \|K\|)^2 \\ = 4(M + \|K\|)^2 / N. \quad \blacksquare$$

**lemma A.4** *Let  $P$  be a projection on a Hilbert space  $\mathcal{H}$ . Let  $\Phi \in \mathcal{H}$  with  $\|\Phi\| = 1$ . Choose  $s > 0$ . Suppose that  $\mathcal{H}'$  is an auxiliary Hilbert space with  $\Psi_0, \Psi_1, \Psi_2 \in \mathcal{H}'$  normalized and  $\Psi_1$  and  $\Psi_2$  orthogonal.*

*Then there exists a bounded self-adjoint Hamiltonian  $L$  on  $\mathcal{H} \otimes \mathcal{H}'$  such that*

$$\exp(-isL)(\Phi \otimes \Psi_0) = P\Phi \otimes \Psi_1 + (1 - P)\Phi \otimes \Psi_2.$$

*The resulting change of state on  $\mathcal{H}$  is*

$$|\Phi\rangle\langle\Phi| \rightarrow P|\Phi\rangle\langle\Phi|P + (1 - P)|\Phi\rangle\langle\Phi|(1 - P)$$

*and on  $\mathcal{H}'$  is*

$$|\Psi_0\rangle\langle\Psi_0| \rightarrow \|P\Phi\|^2 |\Psi_1\rangle\langle\Psi_1| + \|(1 - P)\Phi\|^2 |\Psi_2\rangle\langle\Psi_2|.$$

*proof* As  $\|P\Phi \otimes \Psi_1 + (1 - P)\Phi \otimes \Psi_2\| = 1$ , this lemma is an immediate consequence of lemma A.1. \blacksquare

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