A Response to Pilot Wave Theory.

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This is a version of the overheads that I prepared for a seminar, given at the Cavendish Laboratory in Cambridge on 6th May 2009. Although my seminar was self-contained, it was a response to a splendid series of eight lectures advocating pilot-wave theory given by Mike Towler. His lectures are available here:

web site: **http://www.tcm.phy.cam.ac.uk/~mdt26/pilot_waves.html**

abstract de Broglie-Bohm pilot-wave theory is an interpretation of quantum mechanics which purports to present us with a real world in which real particles guided by real waves hit real screens. I shall introduce the interpretation and explain why I do not believe it to be an adequate solution to the problem of understanding what quantum theory tells us about reality. Given that it is supposed to be a universal quantum theory, I shall look at how pilot-wave theory copes with describing macroscopic thermal systems and human observers. As a non-relativistic theory, Bohmian mechanics of course has difficulties with genuine physical particles such as quarks and photons, but it also has difficulties with the sort of emergent phenomena which seem to arise naturally in the framework of modern heuristic quantum theory. I shall consider how pilot-wave theory deals with non-locality and I shall ask whether any interpretation of quantum theory can ever be fully compatible both with relativity theory and with realism.

Quantum theory is a theory of
interactions, of states, and of probabilities.

Three interpretations of universal quantum theory.

States and wavefunctions.

Pilot-wave theory.

Immediate questions.

Models of measurement.

Observers.

Non-locality.

What are the particles in your head doing?

Universal quantum theory without wavefunction collapse.

Three ways of interpreting such a theory:

Pilot-wave theory.

Deterministic.

Many-minds theory.

Stochastic law.

Both are realist in that they propose that our possible experiences are determined by truths about reality independent of what we want, of what we know, and of what we can verify.

The opposite of realism is waffle.

The conventional modern approach

Heuristic. Calculates probabilities by applying the Born rule to estimated projections on estimated states with no underlying foundation.

Assumes quantum states.

Finds Bohr incomprehensible and debate about incompatible properties unnecessary.

Looks at how quantum states behave to justify separate analysis for each separate experiment (contextuality).

Takes for granted the existence of observers.

Many-worlds is implicit.

Universal quantum theory without wavefunction collapse.

Macroscopic objects have quantum states.

These are restrictions of some (unchanging) global quantum state.

Quantum systems: the air in this room, your brain.

Restricted states at the macroscopic level are (almost) certainly not pure.

$$S(\rho) = -\mathrm{tr}(\rho \log \rho).$$

If $\rho = \sum_{m=1}^{M} p_m |\psi_m><\psi_m|$, then $S(\rho) \leq \log M$.

Proof:

$$0 \leq \mathrm{tr}(\rho \log \rho - \rho \log \tau)$$

where $\tau = \sum_{m=1}^{M} 1/M |\psi_m><\psi_m|$.

e.g. $\rho = |\psi><\psi| \Rightarrow S(\rho) = 0.$

A glass of water – thermodyamical entropy $k_B \log(10^{3\times 10^{25}})$ – cannot be modelled by a quantum state with a decomposition into fewer than $10^{3\times 10^{25}}$ orthogonal pure states.

These states are not ensembles:

On a typical subspace of a large Hilbert space, the restriction of a generic pure state will have maximal entropy.

The global quantum state may (or may not) be pure.

We cannot tell.

Local $|\psi><\psi|$

global $|\psi><\psi| \otimes \sum_j r_j|\varphi_j><\varphi_j|$

or local $\sum_{m=1}^{M} p_m|\psi_m><\psi_m|$

and global

$$\sum_{m=1}^{M} \sqrt{p_m}|\psi_m \otimes \varphi_m> \sum_{m'=1}^{M} \sqrt{p_{m'}}<\psi_{m'} \otimes \varphi_{m'}|.$$

Bohm's theory says that a universe with N particles can be described at any time t by giving a wavefunction

$$\Psi = \Psi(x_1, \ldots, x_N, t) \in L^2(\mathbb{R}^{3N})$$

and a position $X_n = X_n(t) \in \mathbb{R}^3$ for each particle $(n = 1, \ldots, N)$.

Ψ satisfies the non-relativistic Schrödinger equation

$$i\hbar \frac{\partial \Psi}{\partial t} = -\sum_{r=1}^{N} \frac{\hbar^2}{2m_r} \nabla_r^2 \Psi + V\Psi.$$

The velocity of particle r is given by the r-gradient of the phase of $\Psi = Re^{iS/\hbar}$ at $X(t) = (X_1, X_2, \ldots, X_N)$

$$m_r \dot{X}_r(t) = \nabla_r S(t, x_1, x_2, \ldots, x_N)|_{\mathbf{x}=\mathbf{X}(\mathbf{t})}.$$

The fundamental assumption:

a measurement of the position of particle r tells us the value of X_r and (taking $r = 1$ wlog) the probability density with which we will find value X_1 is

$$\int |\Psi(X_1, x_2, \ldots, x_N)|^2 dx_2 \ldots dx_N.$$

What is special about measurement?

What is "finding a value"?

Any (approximate) discovery of a position anywhere in the universe reveals (something about) the corresponding X_r at that time.

Reveals to whom?

Whose probability density?

What is special about position?

What is V? We need to quantize the electro-magnetic field.

What is a particle?

Is a photon a particle?

Sometimes light is in a particle-like photon state,
sometimes it is in a wave-like coherent state.

Electrons have particle-like states, wave-like Bloch states,
and coherent multi-particle superconducting states.

Heuristic quantum theory and many-minds
theory accept all on the same level.

Pilot-wave theory either pilots particles or waves.

Is a quark a particle?

Is a phonon?

Is a harmonic oscillator excitation?

Is a virtual particle?

A phonon is an emergent phenomenon.

Is an electron emergent/ Like an oscillator
excitation/ A pole in the S-matrix?

What is Ψ? This is supposed to be a theory without collapse, so
presumably we can guess at Ψ by working forward from the big bang.

What changes when we gain information?

Example: Take

$$\Psi = \Psi(x_1, x_2, \ldots, x_N)$$

where x_1 is the co-ordinate of a particle to be measured and
$x_2, \ldots, x_N$ are co-ordinates of particles in the measuring device.

Assume $N \gg 1$.

Suppose at the end of the measurement

$$\Psi = \sum_k a_k \Psi_k(x_1, x_2, \ldots, x_N)$$

where $\Psi_k = 0$ unless $(x_1, x_2, \ldots, x_N) \in O_k \subset \mathbb{R}^{3N}$ and suppose that
the O_k are macroscopically disjoint in the sense, for example,
that just a few of the x_r suffice to specify at most one of the O_k.

For example, suppose that the O_k define
a finite set of distinct pointer positions.

Then, if we see a suitable number of particles of the
measuring device in O_K, we can, For All Practical
Purposes, replace Ψ by the "effective wavefunction" Ψ_K.

In conventional quantum theory, we argue that, for many
values of r, we know that for each pair (k, k') with $k' \neq k$

$$\int \overline{\Psi_{k'}(x_1, x_2, \ldots, x_N)} \Psi_k(x_1, x_2, \ldots, x_N) dx_r = 0$$

and therefore expectations of operators A
on spaces with other co-ordinates will satisfy

$$<\Psi|A|\Psi> = \sum_k |a_k|^2 <\Psi_k|A|\Psi_k>.$$

This means that on all practically measurable operators,
or for localized measurements or observers, $|\Psi><\Psi|$ is
indistinguishable from the mixture $\sum_k |a_k|^2 |\Psi_k><\Psi_k|$ and

given evidence for a value K of k we can replace $|\Psi><\Psi|$ by $|\Psi_K><\Psi_K|$.

The conventional argument is more general, in as far
as disjointness of spatial support is not required for

$$\int \overline{\Psi_{k'}(x_1, x_2, \ldots, x_N)} \Psi_k(x_1, x_2, \ldots, x_N) dx_r = 0.$$

Indeed x_r need not even be a spatial co-ordinate.

Most importantly, the decomposition need not be into pure states.

Thermal states can remain thermal,

we only assume what we know.

Generality raises the possibility of a "preferred basis" problem
in that there may be more than one suitable decomposition of Ψ.

At least heuristically, this problem can be avoided by invoking
Zurek's idea of environmentally-determined "pointer states".

Many-minds solves the preferred basis problem
by fiat in a way compatible with decoherence.

Only stable pointer states are likely to build
into rich complex long-lasting structures.

In pilot-wave theory, we argue that we can replace Ψ by
Ψ_K, because for some values of r, we know enough about X_r
to determine that $(X_1, X_2, \ldots, X_N) \in O_K$ (there is no chance
of $(X_1, X_2, \ldots, X_N)$ being such that $\Psi(X_1, X_2, \ldots, X_N) = 0$)

then the piloting equations for $(X_1, X_2, \ldots, X_N)$ are the same whether the wavefunction is Ψ or Ψ_K because, for $k \neq K$,

$$\Psi_k(X_1, X_2, \ldots, X_N) = \nabla^2 \Psi_k(X_1, X_2, \ldots, X_N) = 0.$$

Indeed $|y - X| < d \Rightarrow \Psi_k(y) = 0.$

In pilot-wave theory, observers do not need to be implicit.

It can be assumed that a brain is an arrangement of some set of particles. We know that arrangement by being it.

Correlations explain "finding a value".

Problem: which set of particles?

In pilot-wave theory, as in conventional physics, observers just are.

In many-minds theory observers are central, and the aim is to provide a "natural" characterization of them in terms of developing patterns of information.

Non-locality.

The conventional approach.

It's all a bit odd, but we can't send signals so it doesn't matter.

On the other hand, the geometry of locality – special and general – is awesome.

Alice and Bob observe a sequence of singlets. Their observations cannot be explained by information carried by the individual singlets.

The observations seem genuinely random and the compatibility between paired events seems incomprehensible.

Buzzwords: contextuality, counterfactual.

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$$

is not manifestly non-relativistic

$$H = -\frac{1}{2m}\nabla^2 + V$$

is.

In pilot-wave theory, each trajectory depends on the other particles' instantaneous positions:

$$m_r \dot{X}_r(t) = \nabla_r S(t, x_1, x_2, \ldots, x_N)|_{\mathbf{x}=\mathbf{X(t)}}$$

where $\Psi = Re^{iS/\hbar}$.

For example, if

$$\Psi(x, y) = ce^{-(a+ib)(x-y)^2 + ik(x-y) - d(x+y)^2}$$

THEN

$$S = -b(x-y)^2 + k(x-y)$$

AND

$$\dot{X} = k - 2b(X-Y), \dot{Y} = -k - 2b(Y-X).$$

A CHANGE IN X WILL INSTANTLY BE SIGNALLED TO $(\dot{Y}, Y)$.

X CAN BE CHANGED, WITHIN A SHORT TIME, BY CHANGES TO Ψ NEAR X.

THIS CAN BE DONE WITHOUT A CHANGE IN Ψ, IN S, OR IN THE EQUATION $\dot{Y} = -k - 2b(Y - X)$ NEAR Y.

IF ALICE CHANGES HER APPARATUS, THEN THE DEVELOPMENT OF Ψ NEAR X WILL CHANGE, AND IN CONSEQUENCE, THE TRAJECTORY OF Y AND THE RESULT SEEN BY BOB MAY CHANGE.

AS LONG AS ALICE AND BOB CAN ONLY "MEASURE" Ψ, ALICE CANNOT SIGNAL TO BOB.

BUT THEN X AND Y CAN ONLY BE KNOWN UP TO THE FUNDAMENTAL ASSUMPTION THAT THEIR DISTRIBUTION IS DETERMINED BY THE CURRENT WAVEFUNCTION.

AWESOME GEOMETRY IS NOT FUNDAMENTAL.

PARTICLES ARE GENUINELY HIDDEN.

IN A "COLLAPSE" THEORY

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow> - |\downarrow\uparrow>)$$

CHANGES INSTANTLY TO $|\uparrow\downarrow>$ WHEN ALICE FINDS $|\uparrow>$. CHANGE IS SIGNALLED AT THE LEVEL OF THE WAVEFUNCTION WHICH IS "HIDDEN" IN THAT ALICE CANNOT CHOOSE HER RESULT AND BOB CANNOT DISCOVER HER MEASUREMENT CHOICE BY CLONING.

IF A WAVEFUNCTION IS JUST OUR KNOWLEDGE, OF WHAT ARE WE MADE?

IN PILOT-WAVE THEORY THE PARTICLES AND THE WAVEFUNCTION ARE BOTH REAL BUT HIDDEN.

THE QUANTUM STATE ALONE.

HYPOTHESES OF LOCAL QUANTUM FIELD THEORY:

IF Λ_A AND Λ_B ARE STRICTLY SPACELIKE SEPARATED AND ρ_A AND ρ_B ARE ARBITRARY STATES LOCAL TO REGIONS Λ_A AND Λ_B THEN THERE EXISTS A GLOBAL STATE ρ WITH RESTRICTION EQUAL TO ρ_A ON Λ_A AND TO ρ_B ON Λ_B.

IF Λ_2 IS IN THE CAUSAL SHADOW OF Λ_1 THEN THE STATE ON Λ_2 IS DETERMINED BY THE STATE ON Λ_1.

Changes in state made by Alice cannot be seen by Bob
until he lies in the causal shadow of those changes.

The many-worlds picture.

Alice and Bob make independent local decisions.

They make all possible decisions.

$$S(\rho) = -\mathrm{tr}(\rho \log \rho).$$

Many-minds gives definitive realities
and probabilities to these possibilities.

Information is not exchanged between Alice and Bob until they meet.

But then, to maintain locality and avoid solipsism, all
their possibilities must have been real before they met.

If pilot-wave and many-minds were
genuinely empirically equivalent, then

either action at a distance happens but we cannot touch its causes
or we have many alternative realities which we cannot reach.

Choice:

theoretical beauty: frame dependence or relativity

place of mind: add-on or central.

At the level of observation, local events
depend only on the local quantum state.

Density matrices have many decompositions:

$$\rho = \sum_i p_i |\psi_i><\psi_i| = \sum_j q_j |\varphi_j><\varphi_j|.$$

For example,

$$|a><a| + |b><b| = \tfrac{1}{2}(|a+b><a+b| + |a-b><a-b|).$$

Let $\Psi(x,y) = \sum_i \sqrt{p_i}\psi_i(x)\xi_i(y)$ and
suppose that the ξ_i have disjoint supports.

Then Y determines i.

Suppose that the ψ_i have support in region A and the ξ
elsewhere. Then the local state in A is $\sum_i p_i |\psi_i><\psi_i|$.

The local reality, according to pilot wave
theory, is one of the ψ_i and none of the φ_j.

Example

$$\psi_1(x) = ce^{-ax^2+ikx}, \psi_2(x) = ce^{-ax^2-ikx}$$

$\dot{X} = \pm k$, X has a Gaussian distribution

$$\varphi_1(x) = c'(\psi_1(x) + \psi_2(x)) = c'' e^{-ax^2} \cos kx,$$

$$\varphi_2(x) = d'(\psi_1(x) - \psi_2(x)) = d'' e^{-ax^2} \sin kx$$

$\dot{X} = 0$, X has a fringed Gaussian distribution.

Everett:

Suppose an observer in region A has possible wavefunctions φ_j^1 corresponding to observations φ_j^2 and set $\varphi_j = \varphi_j^1 \otimes \varphi_j^2$.

The φ_j^1 are determined by the structure of the observer and the φ_j^2 by the structure of his observation.

Many-minds does not require the φ_j^1 to be wavefunctions.

Structure only needs to be possible to come into existence.

Possible structure develops from possible structure in an abstract local history.

The global wavefunction could be the vacuum.

"In pilot-wave theory ... It can be assumed that a brain is an arrangement of some set of particles. We know that arrangement by being it."

This assumes that

the actual pilot-wave theory arrangement of particles

determined only by the initial wave-function and the initial arrangement of particles

happens

in the perpetual infoldings of the global wavefunction

to have produced a brain-like arrangement

and that arrangement continues to behave in a brain-like way.

Local mixing mixes the local properties of the global wavefunction and the Bohmian dynamics.

The assumption that there is a "natural" local wavefunction has not been demonstrated.

The assumption that the particle trajectories are "natural" is implausible.

Surreal trajectories.

Particles do not cross nodes.

In a real eigenfunction, the particles are stationary.

In a vacuum, there are no particles, but there are possibilities.

Summary:

we need to do better than heuristic quantum theory

Bohmian mechanics is deterministic and
explains the double slit mystery but

fails to say what a particle or a mind is

ignores the broad freedom of modern physics

violates relativity

is tied to its initial conditions

may not predict our existence

many-minds

does better

but

Taking a many-minds interpretation seriously yields:

a very complicated theory:

"[Donald's] attempt to place on a rational footing the [. . .] 'many-minds' idea — that each stream of consciousness is a mere individual branch of a tree of diverging and contradictory streams of consciousness, all equally real in the large objective sense — appears, in view of its intricate ad hoc nature, to be more like the death rattle of a collapsing radical idea than the foundation of a viable theory of natural reality." (Stapp 2004)